Severe Edge Effects and Simple Complimentary Interior Solutions for Thin-Walled Anisotropic and Composite Structures

FINAL REPORT

C. O. Horgan and J. G. Simmonds

December, 1994

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University of Virginia
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C. O. Horgan and J. G. Simmonds

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School of Engineering & Applied Sciences
University of Virginia
Charlottesville, VA 22901

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13. ABSTRACT (*Maximum 200 words*)

Many useful thin-walled structures of interest to the U. S. Army, such as rifle barrels, automotive parts, rocket casings, helicopter blades, driveshafts, and containment vessels, are often constructed of layers of anisotropic, filament or fiber-reinforced materials. While many of these structures are subject to severe mechanical, inertial, or thermal loads, they often must be designed to remain elastic. This means that it is particularly important to be able to compute accurately global characteristics, such as buckling loads and natural frequencies, as well as local information such as stresses near holes or edges. Two important, complementary regions of such structures, have been studied, namely, the *interior* where there are no steep stress gradients, and the edge zone(s) where stress gradients are high. For both regions, simplified, cost-effective asymptotic methods have been developed. These considerations are particularly important in layered, anisotropic structures because many investigators have (1) claimed that higher-order (and hence computationally expensive) beam, plate, or shell theories are needed for such structures and (2) not paid sufficient attention to the particularly severe end effects (breakdown of Saint-Venant's principle) such structures engender.

14. SUBJECT TERMS
end or edge effects; composite structures; Saint-Venant's principle; asymptotic methods; stress decay rates; decay lengths; linear and nonlinear elasticity

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ABSTRACT

Many useful thin-walled structures of interest to the U. S. Army, such as rifle barrels, automotive parts, rocket casings, helicopter blades, driveshafts, and containment vessels, are often constructed of layers of anisotropic, filament or fiber-reinforced materials. While many of these structures are subject to severe mechanical, inertial, or thermal loads, they often must be designed to remain elastic. This means that it is particularly important to be able to compute accurately global characteristics, such as buckling loads and natural frequencies, as well as local information such as stresses near holes or edges. Two important, complementary regions of such structures, have been studied, namely, the *interior* where there are no steep stress gradients, and the edge zone(s) where stress gradients are high. For both regions, simplified, cost-effective asymptotic methods have been developed. These considerations are particularly important in layered, anisotropic structures because many investigators have (1) claimed that higher-order (and hence computationally expensive) beam, plate, or shell theories are needed for such structures and (2) not paid sufficient attention to the particularly severe end effects (breakdown of Saint-Venant's principle) such structures engender.

FINAL REPORT

STATEMENT OF PROBLEM STUDIED:

Thin-walled structures of interest to the U. S. Army, such as rifle barrels, rocket casings, helicopter blades, and containment vessels, are often constructed of layers of anisotropic, filament or fiber-reinforced materials which must be designed to remain elastic. This means that it is particularly important to be able to compute accurately global characteristics, such as buckling loads and natural frequencies, as well as local information such as stresses near holes or edges. Our work concerns the study of two important, complementary regions of such structures, namely, the *interior* where there are no steep stress gradients, and the edge zone(s) where stress gradients are high. For both regions, we continue to develop simplified, cost-effective asymptotic methods. These considerations are particularly important in layered, anisotropic structures because many investigators have (1) claimed that higher-order (and hence computationally expensive) beam, plate, or shell theories are needed for such structures and (2) not paid sufficient attention to the particularly severe end effects (breakdown of Saint-Venant's principle) such structures engender.

Summary of most important results:

For convenience, we refer to the list of publications which follows and summarize the most significant results obtained therein.

1. Horgan, C. O. and Simmonds, J. G., "Asymptotic Analysis of an End-Loaded Transversely Isotropic Elastic Semi-Infinite Strip Weak in Shear," *International Journal of Solids and Structures* **27** (1991), 1895-1914.

Abstract. We use linear elasticity to study a transversely isotropic (or specially orthotropic), semi-infinite slab in plane strain, free of traction on its faces and at infinity and subject to edge loads or displacements that produce stresses and displacements that decay in the axial direction. The governing equations (which are identical to those for a strip in plane stress, free of traction on its long sides and at infinity, and subject to tractions or displacements on its short side) are reduced, in the standard way, to a fourth order partial differential equation with boundary conditions for a dimensionless Airy stress function, f. We study the asymptotic solutions to this equation for four sets of end conditions—traction, mixed, displacement—as ε, the ratio of the shear modulus to the geometric mean of the axial and transverse extensional moduli, approaches zero. In all cases, the solutions for f consist of a "wide" boundary layer that decays slowly in the axial direction (over a distance that is long compared to the width of the strip) plus a "narrow" boundary layer that decays rapidly in the axial direction (over a distance that is short compared to the width of the strip). Moreover, we find that the narrow boundary layer has a "sinuous" part that varies rapidly in the transverse direction, but which, to lowest order, does not enter the boundary conditions nor affect the transverse normal stress or the displacements. Because the exact bi-orthogonality condition for the eigenfunctions associated with f can be replaced by simpler orthogonality conditions in the limit as $\varepsilon \to 0$, we are able to obtain, to lowest-order, explicit formulas for the coefficients in the eigenfunction expansions of f for the four different end conditions.

2. Mehrabadi, M. M., Cowin, S. C. and Horgan, C. O., "Strain Energy Density Bounds for Linear Anisotropic Elastic Materials," *Journal of Elasticity* **30** (1993), 191-196.

Abstract. We discuss the problem of obtaining upper and lower bounds for the strain-energy density in linear anisotropic elastic materials. One set of bounds is given in terms of the *magnitude of the stress field,* another in terms of the *magnitude of the strain field.* Results of this kind play a major role in the analysis of Saint-Venant's Principle for anisotropic materials and structures. They are also useful in estimating global quantities such as total energies, buckling loads, and natural frequencies. For several classes of elastic symmetry (e.g., cubic, transversely isotropic, hexagonal, and tetragonal symmetry) the *optimal constants* appearing in these bounds are given *explicitly* in terms of the elastic constants. This makes the results directly accessible to the design engineer. Such explicit results are rare in the field of anisotropic elasticity. For more elaborate symmetries (e.g., orthotropic, monoclinic, and triclinic) the optimal constants depend on the solution of cubic and sextic equations, respectively.

3. Horgan, C. O. and Payne, L. E., "A Saint-Venant Principle for a theory of nonlinear plane elasticity," *Quarterly of Applied Mathematics* **50** (1992), 641-675.

Abstract. The effect of *nonlinearity* on the decay of end effects is examined within the context of a theory of plane strain for isotropic materials. The model takes material nonlinearity into account while restricting attention to small displacement gradients. An estimate for the decay of Saint-Venant end effects is obtained (in terms of the load level, domain geometry, and material properties) for a rectangular region subjected to end loads only. The results predict a progressively *slower* decay of end effects with increasing load level. Thus, a *wide* boundary-layer phenomenon is also obtained in this problem, but now due to *nonlinearity.* Such effects are ever more important to understand in view of the current technological demands to improve the performance of materials and structures.

4. Horgan, C. O., and Payne, L. E., "The Effect of Constitutive Law Perturbations on Finite Anti-Plane Shear Deformations of a Semi-Infinite Strip," *Quarterly of Applied Mathematics* **51** (1993),441-465.

Abstract. This paper is concerned with assessing the effects of small perturbations in the constitutive laws on anti-plane shear deformations fields arising in the theory of nonlinear elasticity. The mathematical problem is governed by a second-order quasilinear partial differential equation in divergence form. Dirichlet (or Neumann) boundary value problems on a semi-infinite strip, with non-zero data on one end only, are considered. Such problems arise in investigation of Saint-Venant end effects in elasticity theory. The main result established provides a comparison between two solutions, one of which is a solution to a simpler equation, for example Laplace's equation. Three examples involving perturbation of power-law material models are used to illustrate the results. The results obtained are of interest in *constitutive modeling of material behavior*. Thus errors made in constructing a specific constitutive model are correlated with corresponding errors in the solution of boundary-value problems. Such results can be used by designers to assess the robustness of particular constitutive models.

5. Duva, J. M. and Simmonds, J. G., "The Usefulness of Elementary Theory for the Linear Vibrations of Layered, Orthotropic Elastic Beams and Corrections due to Two-Dimensional End Effects," *Journal of Applied Mechanics* **58** (1991), 175-180.

Abstract With the aid of formal asymptotic expansions, we conclude not only that elementary (Euler-Bernoulli) beam theory can be applied successfully to layered, orthotropic beams, possibly weak in shear, but also that, in computing the lower natural frequencies of a cantilevered beam, the most important correction to the elementary theory—of the relative order of magnitude of the ratio of depth to length—comes from effects in a neighborhood of the built-in end. We compute this correction using the fundamental work on semi-infinite elastic strips of Gregory & Gladwell (1982) and Gregory & Wan (1984). We also show that, except in unusual cases (e.g., a zero Poisson's ratio in a homogeneous, elastically isotropic beam), Timoshenko beam theory produces an erroneous correction to the frequencies of elementary theory of the relative order of magnitude of the square of the ratio of depth to length.

6. Duva, J. M. and Simmonds, J. G., "The Influence of Two-Dimensional End Effects on the Natural Frequencies of Cantilevered Beams Weak in Shear," *Journal of Applied Mechanics* **59** (1992), 230-232.

Abstract. This Brief Note supplies further specific evidence to support a thesis that we—and others—have asserted before:

a. In elastically orthotropic beams (that may be either homogeneous or layered), so-called higher-order theories offer, in general, no advantages unless two-dimensional end effects are considered. Moreover,

b. if end effects are considered, then simple, easily computable refinements to elementary theory (properly interpreted and applied) suffice.

7. England, R. and Simmonds, J. G., "Simplifications under the Kirchhoff Hypothesis of Taber's Nonlinear Theory for the Axisymmetric Bending and Torsion of Elastic Shells of Revolution," *International Journal of Solids and Structures* **28** (1991), 507-515.

Abstract We show that, under the Kirchhoff hypothesis, Taber's recent theory for the simultaneous axisymmetric bending and torsion of shells of revolution undergoing large strains can be simplified considerably. In general, his 33 equations can be reduced to four first-order ordinary differential equations and two algebraic equations for six unknowns. For small strains, the equations can be reduced further to two coupled nonlinear equations for the meridional angle of rotation and a stress function, as in Reissner's theory of torsionless, axisymmetric deformation.

8. Simmonds, J. G., "An Asymptotic Analysis of End Effects in the Axisymmetric Deformation of Elastic Tubes Weak in Shear: Higher-Order Shell Theories are Inadequate and Unnecessary," *International Journal of Solids and Structures* **29** (1992), 2441-2461.

Abstract This paper specializes to a semi-infinite tube Horgan's (1974) two-stress-function formulation of the equations for the axisymmetric deformation of a linearly elastic transversely isotropic cylindrical body free of surface tractions. The ratio of the tube's shear modulus to its radial (transverse) extensional modulus is taken to be of the order of magnitude of the square root of its thickness to its mean radius. The equations are solved by formal asymptotic expansion in (fractional) power s of the thickness to radius ratio for four canonical sets of end conditions: (A) axisymmetric equilibrated tractions; (B) and (C), two different combinations of tractions and displacements; and (D) axisymmetric radial and axial displacements. The solutions exhibit interior (i.e., shell-like) parts and wide and narrow boundary (or edge) layers, the latter containing components that vary extremely rapidly through the thickness of the tube. The

analysis focuses on computing the lowest-order correction, both in the interior and in the boundary layers, to classical shell theory. It is shown that in cases (A)-(C) the interior correction to classical shell theory—that is, those effects so-called higher-order shell theories attempt to capture—can (ultimately) be determined directly, in terms of the edge data, but that in case of prescribed displacements (D), the computation of (three-dimensional) boundary-layer effects is essential. These conclusions are consistent with those for elastically *isotropic* shells found by Gregory & Wan (1992) who used ingenious arguments based on the Betti Reciprocity Principle.

9. Crafter, E. C., Heise, R. M., Horgan, C. O., and Simmonds, J. G., "The Eigenvalues for a Self-Equilibrated, Semi-Infinite, Anisotropic Elastic Strip," *Journal of Applied Mechanics* **60** (1993), 276-281.

Abstract The linear theory of elasticity is used to study an homogeneous anisotropic semi-infinite strip, free of tractions on its long sides and subject to edge loads or displacements that produce stresses that decay in the axial direction. If one seeks solutions for the (dimensionless) Airy stress function of the form $\phi = e^{-\gamma x} F(y)$, γ constant, then one is led to a fourth-order eigenvalue problem for $F(y)$ with complex eigenvalues γ. This problem, considered previously by Choi & Horgan (1977), is the anisotropic analog of the eigenvalue problem for the Fadle-Papkovich eigenfunctions arising in the isotropic case. The decay rate for Saint-Venant end effects is given by the eigenvalue with smallest positive real part. For an isotropic strip, where the material is described by two elastic constants (Young's modulus and Poisson's ratio), the associated eigencondition is *independent* of these constants. For transversely isotropic (or specially orthotropic) materials, described by *four* elastic constants, the eigencondition depends only on *one* elastic parameter. Here, we treat the fully anisotropic strip described by *six* elastic constants and show that the eigencondition depends on only *two* elastic parameters. Tables and graphs for a scaled complex-valued eigenvalue are presented. These data allow one to determine the Saint-Venant decay length for the *fully anisotropic* strip, as we illustrate by a numerical example for an end-loaded off-axis graphite-epoxy strip.

10. Simmonds, J. G., "Simplified Proofs of Three Theorems on the Kinematics of Axisymmetric Deformation of Shells of Revolution," *Quarterly of Applied Mathematics* **52** (1994), 283-287.

Abstract. It is shown that certain *a priori* estimates on the magnitude of the displacement and strain gradients in general shells of revolution undergoing axisymmetric deformation in which the strains are small but the meridional rotations arbitrarily large are a simple consequence of a nonlinear compatibility condition derived by Reissner over 40 years ago and some elementary geometrical inequalities.

11. Miller, K. L. and Horgan, C. O., "Conservation Properties for Plane Deformations of Isotropic and Anisotropic Linearly Elastic Strips," *Journal of Elasticity* **33** (1993), 311-318.

Abstract. Plane deformations of a rectangular strip, composed of an homogeneous fully anisotropic linearly elastic material, are considered. The strip is in equilibrium under the action of end loads, with the lateral sides traction-free. Two conservation properties for certain cross-sectional stress measures are established, generalizing previously known results for the isotropic case. It is noteworthy that in the first of these conservation laws only one of the off-axis elastic constants appears explicitly while in the second only the opposite off-axis constant appears

explicitly. Such conservation properties are useful in assessing the influence of material anisotropy on Saint-Venant's principle, as well as in establishing convexity properties for cross-sectional stress measures. In particular, it is anticipated that the results should be useful in determining the extent of edge effects in the off-axis testing of anisotropic and composite materials.

12. Polignone, D. A. and Horgan, C. O., "Cavitation for incompressible anisotropic nonlinearly elastic spheres," *Journal of Elasticity* **33** (1993) 27-65.

Abstract. Here we examine the *effects of anisotropy* in a large deformation nonlinear elasticity problem. Compared to the linear theory, very little is known about such effects in the nonlinear theory. The problem considered concerns a solid sphere, composed of an incompressible homogeneous nonlinearly elastic material which is transversely isotropic about the radial direction. The sphere is subjected to a uniform radial tensile dead load. At sufficiently large loads, a branch of radially symmetric configurations involving a traction-free internal cavity bifurcates from the undeformed configuration. Such solutions describe the phenomenon of "cavitation" in solids. They may also be interpreted in terms of the *rapid growth of a pre-existing microvoid* at sufficiently large loads. Such phenomena serve as precursors to fracture and play a fundamental role in understanding failure mechanisms in solids. It is shown that *anisotropy* gives rise to a "snap cavitation" instability where the cavity has finite radius on first appearance. This is reminiscent of the snap-through buckling phenomenon of structural mechanics.

13. Horgan, C. O. and Payne, L. E., "Phragmén-Lindelöf type results for harmonic functions with nonlinear boundary conditions," *Archive for Rational Mechanics and Analysis* **122** (1993), 123-144.

Abstract. We are concerned with investigating the asymptotic behavior of harmonic functions defined on a three-dimensional semi-infinite cylinder, where homogeneous nonlinear boundary conditions are imposed on the lateral surface of the cylinder. Such problems arise in the theory of steady-state heat conduction. The classical Phragmén-Lindelöf theorem states that harmonic functions which vanish on the lateral surface of the cylinder must either grow exponentially or decay exponentially with distance from the finite end of the cylinder. Here we show that the results are significantly different when the homogeneous Dirichlet boundary condition is replaced by the nonlinear heat-loss or heat-gain type boundary condition. We show that *polynomial growth (or decay)* or *exponential growth (or decay)* may occur, depending on the form of the nonlinearity. Explicit estimates for the growth or decay are obtained.

14. Horgan, C. O. and Payne, L. E., "On the asymptotic behavior of solutions of linear second-order boundary-value problems on a semi-infinite strip," *Archive for Rational Mechanics and Analysis* **124** (1993), 277-303.

Abstract. This paper is concerned with investigating the asymptotic behavior of solutions of a linear second-order variable coefficient elliptic partial differential equation in divergence form defined on a two-dimensional semi-infinite strip. Such problems arise in the theory of steady-state heat conduction for inhomogeneous anisotropic materials as well as in the theory of anti-plane shear deformations for a linearized inhomogeneous anisotropic elastic solid. For solutions of such equations which vanish on the long sides of the strip, a theorem of Phragmén-Lindelöf type is established providing estimates for the rate of growth or decay which are optimal for the case of constant coefficients. The results are illustrated by several examples. The estimates

obtained in this paper can be used to assess the influence of inhomogeneity and anisotropy on the decay of end effects arising in connection with Saint-Venant's principle.

15. Polignone, D. A., and Horgan, C. O., "Effects of material anisotropy and inhomogeneity on cavitation for composite incompressible anisotropic nonlinearly elastic spheres," *International Journal of Solids and Structures* **30** (1993), 3381-3416.

Abstract. The effects of *material anisotropy* and *inhomogeneity* on void nucleation and growth in incompressible anisotropic nonlinearly elastic solids are examined. A bifurcation problem is considered for a composite sphere composed of two arbitrary homogeneous incompressible nonlinearly elastic materials which are transversely isotropic about the radial direction and perfectly bonded across a spherical interface. Under a uniform radial tensile dead-load, a branch of radially symmetric configurations involving a traction-free internal cavity bifurcates from the undeformed configuration at sufficiently large loads. Several types of bifurcation are found to occur. Explicit conditions determining the type of bifurcation are established for the general transversely isotropic composite sphere. In particular, if each phase is described by an explicit material model which may be viewed as a generalization of the classic neo-Hookean model to anisotropic materials, phenomena which were not observed for the homogeneous anisotropic sphere nor for the composite neo-Hookean sphere may occur. The stress distribution as well as the possible role of cavitation in preventing interface debonding are also examined for the general composite sphere.

16. Horgan, C. O., and Simmonds, J. G., "End effects in anisotropic and composite structures," *Proceedings of Army Symposium on Solid Mechanics,* Plymouth MA (1994), pp. 567-578, (ed. by F. D. Bartlett, Jr., S. C. Chou, K. Iyer and T. W. Wright).

17. Horgan, C. O., and Simmonds, J. G., "Saint-Venant end effects in composite structures," *Composites Engineering* **3** (1994), 279-286.

Abstracts: In these papers, we return to the issue of slow stress diffusion and Saint-Venant end effects in composite structures. Thin-walled structures such as aircraft and automotive parts, rocket casings, helicopter blades and containment vessels are often constructed of layers of anisotropic, filament or fiber-reinforced materials which must be designed to remain elastic. The extent to which *local* stresses, such as those produced by fasteners and at joints, can penetrate girders, beams, plates and shells must be understood by the designer. Thus a distinction must be made between *global* structural elements (where Strength of Materials or other approximate theories may be used) and *local* elements which require more detailed (and more costly) analyses based on exact elasticity. The neglect of end effects is usually justified by appeals to some form of Saint-Venant's principle and years of experience with *homogeneous isotropic elastic structures* has served to establish this standard procedure. Saint-Venant's principle also is the fundamental basis for static mechanical tests of material properties. Thus property measurements are made in a suitable *gage section* where *uniform* stress and strain states are induced and local effects due to clamping of the specimen are neglected on invoking Saint-Venant's principle. Such traditional applications of Saint-Venant's principle require major modifications when strongly anisotropic and composite materials are of concern. For such materials, local stress effects persist over distances *far greater* than is typical for isotropic metals. In [16], we describe plane elastostatic problems where anisotropy induces such extended Saint-Venant end zones. The paper is a review and a comprehensive list of references is given to original work where details of the analyses may be found. A more comprehensive review, which included three-

dimensional problems, effects of nonlinearity, and dynamic effects, is provided in [17]. The implications of such extended end zones due to anisotropy are far reaching in the proper analysis and design of structures using advanced composite materials (see Fig 1. attached).

18. Simmonds, J. G., and Warne, P. G., "Notes on the nonlinearly elastic Boussinesq problem," *Journal of Elasticity* **34** (1994), 69-82.

Abstract. We examine the effect of a concentrated load on a nonlinearly elastic body in the simplest, nontrivial setting. This is what we call the nonlinearly elastic Boussinesq problem: to find the deformation produced in a homogeneous, isotropic, elastic half space by a point force normal to the undeformed boundary, using the exact equations of elasticity for an incompressible or compressible material. First we derive the governing equations from the Principle of Stationary Potential Energy and then we examine some of the implications of the conservation laws of elastostatics when applied to the entire half space, assuming that the well-known linear Boussinesq solution is valid at large distances from the point load. Next, we hypothesize asymptotic forms for the solutions near the point load and, finally, we seek solutions for two specific materials: an incompressible, generalized neo-Hookean (power-law) material introduced by Knowles and a compressible Blatz-Ko material. We find that the former, if sufficiently stiffer than the conventional neo-Hookean material, can support a finite deflection under the point load, but that the latter cannot.

SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED: C. O. Horgan, J. G. Simmonds (PIs); Eric C. Crafter (Ph.D. student), Sarah C. Baxter (Ph.D. student), Paul G. Warne (Ph.D. student), T. McDevitt (Ph.D. student).

Degrees: Paul G. Warne (Ph.D. degree); T. McDevitt (Masters of Applied Mathematics); Kristin L. Miller (Ph.D. degree); Debra A. Polignone (Ph.D. degree); Eric C. Crafter (Masters of Applied Mathematics).

Severe Edge Effects and Simple Complimentary Interior Solutions for Thin-Walled Anisotropic and Composite Structures

C. O. Horgan and J. G. Simmonds
School of Engineering & Applied Science
University of Virginia

Research Objective:

Thin-walled structures of interest to the U.S. Army, such as rifle barrels, rocket casings, helicopter blades, and containment vessels, are often constructed of layers of anisotropic, filament or fiber-reinforced materials which must be designed to remain elastic. This means that it is particularly important to be able to compute accurately global characteristics, such as buckling loads and natural frequencies, as well as local information such as stresses near holes or edges. Our work concerns the study of two important, complementary regions of such structures, namely, the *interior* where there are no steep stress gradients, and the edge zone(s) where stress gradients are high. For both regions, we continue to develop simplified, cost-effective asymptotic methods. These considerations are particularly important in layered, anisotropic structures because many investigators have (1) claimed that higher-order (and hence computationally expensive) beam, plate, or shell theories are needed for such structures and (2) not paid sufficient attention to the particularly severe end effects (breakdown of Saint-Venant's principle) such structures engender.

Approach:

The foregoing issues are examined using the theory of linear elasticity for anisotropic and composite materials. The problems are modeled as boundary-value problems for partial differential equations. Both analytical and numerical techniques are used. The results are compared with experiments. Some nonlinear problems have also been investigated.

Accomplishments:

Several fundamental problems have been solved. The major conclusions are *that end or edge effects are much more significant in composites than in isotropic materials*. Routine invocation of Saint-Venant's principle is *not* justified in the case of anisotropic and composite materials. For example, for isotropic materials in plane deformation, end effects decay over *one* strip width, while for a graphite/epoxy composite, the edge zone is about *four* strip widths. Asymptotic formulas immediately accessible to designers have been obtained which agree remarkably well with experiment and computation.

Significance:

The results have widespread application to structural mechanics issues such as assessing end constraint effects in mechanical testing, determining the influence of fasteners, joints, cut-outs, etc., in composite structures and in the evaluation of the limits of strength-or-materials formulas when applied to composites. Quantitative information of the type provided by this research is crucial to the safe and efficient design of structures using advanced composite materials. The structural integrity of Army components relys crucially on the understanding of edge effects provided by this work.

Edge Effects in Composite Structures

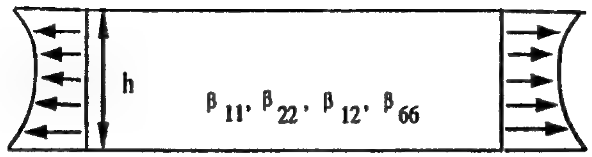
C. O. Horgan and J. G. Simmonds
School of Engineering & Applied Science
University of Virginia

Assumptions

I. Orthotropic material

- 2-D plane stress/strain
- Linear elasticity

(extension problem)



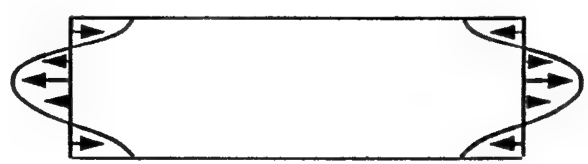
II. Statically equivalent loading

(same resultant force and moment)

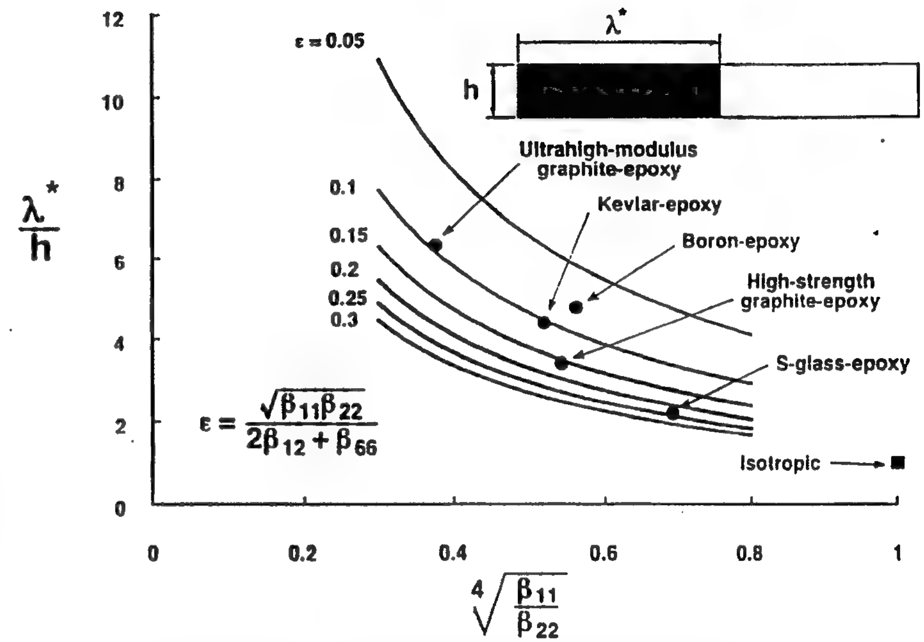
III. Self-equilibrated end loads

- The stress fields decay from the ends
- Rate of decay can be found
- Characteristic decay lengths

(Superposition of I and II)



Results



Decay length λ^* for semi-infinite orthotropic strips subjected to self-equilibrated end loads.

A comparison between several commonly used composite materials is illustrated for specially orthotropic materials. The characteristic decay length (i.e., the distance over which end effects decay to 1% of their end values) versus a nondimensional material parameter is plotted and the results for various materials are indicated by the dots shown on the curves. The decay length for an isotropic material is shown by the dark square. It is seen that the latter has the *smallest* decay length and that this is approximately equal to the width of the strip. This figure can be used directly in the design process to account for anisotropic end effects.

ARMY CONTACT

(i) Continued close contact maintained with Dr. Gary L. Anderson.

(ii) Supplementary information frequently sent to Dr. Anderson for the Annual Internal Review of the Engineering Sciences Division Structures & Dynamics Program.

(iii) Letters and reprints sent to several Army laboratories (see attached list).

(iv) Visited Dr. F. Bartlett, Army Aerostructures Directorate, NASA Langley Research Center, Hampton, VA, 6 January, 1993.

(v) Met with several Army personnel at Army Symposium on Solid Mechanics, Plymouth, MA, 17-19 August, 1993. (Paper presented by J. G. Simmonds.)

(vi) Met with Army personnel at ARO Workshop on "Dynamic Response of Composite Structures," New Orleans, 29-31 August, 1993. (Keynote address presented by C. O. Horgan.)

(vii) Met with Colonel Donald Tarter, deputy commander of the U.S. Army Recruiting Brigade, (Atlanta) at recruiting meeting, University of Virginia, February 24, 1994. Interacted with several other Army personnel at this meeting.

(viii) Letters of interest sent previously from Army engineers and scientists.

Technology Transfer

The results of this research are being widely utilized in the technical literature on composite materials, with technology transfer to such areas as composite design and materials testing. The attached letter from Dr. W. B. Avery of the Boeing Commercial Airplane Group attests to the applicability of the results. Dr. Horgan visited the Boeing Company in Seattle on July 1, 1994 to further this important area of technology transfer. We have also interacted extensively with Dr. M. P. Nemeth, NASA Langley, Structural Mechanics Division. Dr. Nemeth served as a committee member for the Ph.D. dissertation defense of Ms. Kristin L. Miller on April 25, 1994. Her dissertation was entitled: "End Effects for Plane Deformation of an Elastic Anisotropic Semi-Infinite Strip." Dr. Miller was offered an appointment as NRC Postdoctoral Research Associate, National Institute for Standards & Technology (NIST), Gaithersburg, MD. NIST scientists are extremely interested in applications of our research. She was heavily recruited by the National Security Agency (NSA) and decided to accept an appointment as Cryptologic Mathematician, NSA, Fort Meade, MD, Sept. 1994. The results of our research are also regularly sent to AFOSR (Dr. A. Nachman, Program Director, Applied Analysis) and to Air Force personnel at Wright Laboratories, Dayton, Ohio. Prof. Horgan visited Air Force scientists in Flight Dynamics Directorate, Wright Labs, and presented a colloquium lecture on August 11, 1994.

A Technology Transfer Example

A Boeing/NASA Advanced Technology Composite Aircraft Structures (ATCAS) Program has been active since 1989. The primary objective of this program is to:

> "Develop an integrated technology (manufacturing & structures) and demonstrate a confidence level that permits cost-and weight-effective use of advanced composite materials in primary structures of aircraft with the emphasis on pressurized fuselages."

In this program, a section of a widebody aircraft (244" dia) just aft of the wing/body intersection is being analyzed by the Boeing Commercial Airplane Group in Seattle, Washington. Sandwich structures are being used for the side and keel of this section. Compression testing of laminate coupons indicate the need to incorporate Saint-Venant end effects in interpretation of the test data. The work of the PI's is being utilized in this effort. One of the P.I's (C. O. H.) visited the Boeing Group in Seattle on July 1, 1994 to consolidate this interaction. It is planned to engage in collaborative research with the Boeing scientists (Dr. W. A. Avery, coordinator). One objective is to develop a systematic testing program to be carried out by Integrated Technologies, Inc. (Intec), Bothell, WA, under subcontract to Boeing. Preliminary tests by Intec have indicated problems due to end effects in the sandwich panels under investigation. It is anticipated that the theoretical results obtained in our research program will have direct application to these problems. In fact, the interaction with the Boeing/Intec group is providing additional motivation and stimulus to our efforts in understanding the extent of Saint-Venant end effects in advanced composite materials and structures.

The Boeing Company
P.O. Box 3707
Seattle, WA 98124-2207

December 20 1993
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Professor Cornelius Horgan
Department of Applied Mathematics
University of Virginia
Thornton Hall
Charlottesville, VA 22903-2442

BOEING

Dear Professor Horgan,

I am writing to you in response to your letter of November 12, 1993 to Scott Finn. Thank you for sending copies of your papers on St. Venant effects. Mike Nemeth is correct in his perception that Boeing is interested in St. Venant effects in composite structures. In fact, I independently searched the literature for papers on St. Venant effects and came up with many authored by you and your colleagues. I would like to take this opportunity to give you a brief background on our project and then describe some of the technical issues that I think might interest you.

Boeing is currently funded under NASA's Advanced Composites Technology (ACT) program to develop the materials, structures, and manufacturing technology necessary to build a fuselage section for a widebody commercial transport. Boeing calls its program the Advanced Technology Composite Aircraft Structure (ATCAS) program. In this program we have chosen for study a section of a widebody aircraft (244" dia.) just aft of the wing/body intersection. The section is approximately 32 feet long. This section is chosen because its geometry presents most of the technical challenges of producing a composite fuselage structure. It contains landing gear cutouts, cargo door cutouts, window cutouts, and complex loading due to its location with respect to the wing. The program started in May of 1989 and Phase B is scheduled to conclude in 1995. During these 2 phases we have designed, built and tested several subcomponent composite structures. Most of the early work concentrated on the fuselage crown, which is a skin/stringer design with relatively thin skins. More recently, we have moved on to the keel and side structure. These structure are much more difficult to design due to the presence of a complex load state and the number of cutouts. In 1995 Phase C is scheduled to begin. During Phase C Boeing will be contracted to build a full barrel Section 46.

Although we chose skin/stringer as the structure type for the crown, we have chosen sandwich structure for the side and keel. We are using Hercules' AS4/8552 for the skin and Hexcel's HRP honeycomb core. In the past few months we have been conducting compression tests of sandwich structure with impact damage, notches, and undamaged. All of the specimens have been instrumented with strain gages and on some we have collected photoelastic data.

As you are probably aware it is common practice to use relatively short coupons for compression tests in order to prevent global buckling. Boeing's compression-after-impact (CAI) coupon is 6" x 4", which presents an aspect ratio L/W which is only 1.5. During our program we conducted a compression test of a sandwich specimen with a 1" hole. The dimensions of that coupon were 18" x 12". We did not observe the hole size strength effect we expected and came to the conclusion that the load wasn't evenly introduced into the specimen; hence, the hole never saw the expected stress concentration. In another experiment we performed compression testing of 5" x 7" and 5" x 12" open hole (1") solid laminate coupons. Photoelastic data was collected for both specimens. The results showed that better load introduction was obtained in the longer specimen. These seem to point to St. Venant effects and/or specimen fixturing conditions as the reasons for uneven load introduction. In any event, it appears that L/W ratios on the order of 1.5 are insufficient for credible test data. We have somewhat arbitrarily tried to use L/W rations of at least 2 on this program.

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I would be most interested in having some informal discussions with you on these issues when you visit Seattle next year. Perhaps you can give us some insight on St. Venant effects in sandwich composite structure, and maybe you might leave with some ideas for future research. I'm looking forward to meeting you.

Sincerely,

William B. Avery, Ph.D
M/S 6H-PJ
206-234-0444